Neutrino masses and mixing parameters in a left-right model with mirror fermions

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Abstract

In this work we consider a left-right model containing mirror fermions with gauge group $\mathrm{SU}(3)_C\otimes\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R\otimes\mathrm{U}(1)_{Y'}$. The model has several free parameters which here we have calculated by using the recent values for the squared-neutrino mass differences. Lower bound for the mirror vacuum expectation value helped us to obtain crude estimations for some of these parameters. Also we estimate the order of magnitude of the masses of the standard and mirror neutrinos.

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1 Introduction

The understanding of the properties of neutrinos is a problem that has required an immense experimental and theoretical effort. There is now convincing evidence that neutrinos have non-standard properties: they have masses and their flavor states mix, leading to processes like neutrino oscillations.

There are various features that make neutrinos specially interesting. The smallness of neutrino masses is probably related to the fact that neutrinos are completely neutral, i.e., they carry no electric charge which is exactly conserved, and are Majorana particles with masses inversely proportional to the large scale where lepton number (L) conservation is violated. [1] The relation with L non-conservation and the fact that the observed neutrino oscillation frequencies are compatible with a large scale for L non-conservation, establish connection with Grand Unified Theories (GUT's). Then, neutrino masses and mixing offer a different perspective on the problem of flavor and the origin of fermion masses.

Another modern interest in neutrinos arises from their significance in astrophysics and cosmology [2] and the possible non-negligible contribution of neutrinos to hot dark matter in the Universe.

An alternative explanation of small neutrino masses comes from the concept of extra dimensions beyond the three ones that we know [3]. It has been suggested that right-handed neutrinos (but not other particles of the Standard Model) experience one or more of these extra dimensions. The right-handed neutrinos then only spend part of their time in our world, leading to apparently small neutrino masses.

From the experimental study of atmospheric and solar neutrinos we have considerably improved our knowledge. The idea of neutrino oscillations have gained support from the Japanese experiment Super-Kamiokande [4] which in 1998 showed that there was a deficit of muon neutrinos reaching Earth when cosmic rays strike the upper atmosphere. These results were interpreted as muon neutrinos oscillating into tau neutrinos, but they were not detected.

Recently The Sudbury Neutrino Observatory (SNO)[5] in Canada had confirmed that the solar neutrino deficit is due to neutrino oscillations and not to a flaw in the model of the Sun: the total neutrino flux is in agreement with the solar model but only about one third arrives on Earth as ν_e, while the remaining part consists of other kinds of neutrinos, presumably ν_μ and ν_τ.

The KamLAND experiment has established that $\bar{\nu}_e$ from reactors show oscillations over an average distance of about 180 Km which are perfectly compatible with the frequency and mixing angle corresponding to one of the solutions of the solar neutrino problem (the Large Angle solution) [6].

In this paper we use a model with gauge group SU$(3)_C\otimes$SU$(2)_L\otimes$SU$(2)_R$ $\otimes$U$(1)_{Y'}$ which contains mirror fermions. In Section 1 we give an introduction to the problem that we deal with; in Sec. 2 we present the model and discuss the symmetry breaking part with the help of two scalar doublets. Sec. 3 is devoted to write the Yukawa couplings for Majorana and Dirac neutrinos, although we consider only these last ones in calculations; the neutrino mass matrix is shown in terms of three parameters; we diagonalise it and get expressions for the physical neutrino masses. The use of their experimental values and bounds gives us approximate numbers for the involved parameters.

2 The model

The gauge group symmetry is [7] $G = \mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{Y'}$ and the content of fermions assumes the ordinary quarks and leptons including the mirror counterpart, where the leptonic sector is assigned under the group G in the form:

$l_{iL} \sim (1, 2, 1, -1)_L$,

$\nu_{iR} \sim (1, 1, 1, 0)_R$,

$e_{iR} \sim (1, 1, 1, -2)_R$,

$\hat{\nu_{iL}} \sim (1, 1, 1, 0)_L$,

$\hat{e_{iL}} \sim (1, 1, 1, -2)_L$,

$\hat{l_{iR}} \sim (1, 1, 2, -1)_R$,

where i = 1,2,3 is the family index, the numbers in parenthesis are the quantum numbers of the fermionic fields under the groups $\mathrm{SU}(3)_C$, $\mathrm{SU}(2)_L$, $\mathrm{SU}(2)_R$, and $\mathrm{U}(1)_{Y'}$, respectively; the last entry corresponds to the hypercharge (Y'), with the electric charge defined by $Q = \mathrm{T}_{3L} + \mathrm{T}_{3R} + \frac{Y'}{2}$.

The "Spontaneous Symmetry Breaking" (SSB) is proposed to be achieved in the stages

$$\mathrm{G} \longrightarrow \mathrm{G}_{SM} \longrightarrow \mathrm{SU}(3)_C \otimes \mathrm{U}(1)_Q \tag{1}$$

where $\mathrm{G}_{SM} = \mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ is the "Standard Model" group symmetry, and $\frac{Y}{2} = T_{3R} + \frac{Y'}{2}$.

The Higgs sector used to induce the SSB in eq.(1) involves the scalar fields

$$\Phi = (1, 2, 1, 1) \tag{2}$$

$$\hat{\Phi} = (1, 1, 2, 1) \tag{3}$$

where the entries correspond to the transformation properties under the symmetries of the group G, with the "Vacuum Expectation Values" (VEV's)

$$< \Phi >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{4}$$

$$< \hat{\Phi} >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \hat{v} \end{pmatrix} \tag{5}$$

Here the VEV's v and $\hat{v}$ are related to the masses of the charged gauge bosons W and $\hat{W}$ through $M_W = \frac{1}{2} g_L v$; $M_{\hat{W}} = \frac{1}{2} g_R \hat{v}$, with g_L and g_R being the coupling constants of $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$, and $g_L = g_R$ if we demand L-R symmetry.

3 Neutrino mass matrix and mixing parameters

At this point we describe how to obtain the mass terms for the sector of neutrinos (ordinary and mirror).

With the fields of fermions introduced in the model, we may write the gauge invariant Yukawa couplings:

$$h_{ij}\overline{\hat{\nu}_{iL}}\nu_{jR} + \lambda_{ij}\bar{l}_{iL}\tilde{\Phi}\nu_{jR} + \eta_{ji}\hat{l}_{jR}\tilde{\hat{\Phi}}\hat{\nu}_{iL} + h.c., \tag{6}$$

with $\tilde{\Phi} = \mathrm{i}\ \sigma_2\Phi^*$ and $\tilde{\hat{\Phi}} = \mathrm{i}\ \sigma_2\hat{\Phi}^*$, where h_{ij}, λ_{ij} and η_{ij} are Yukawa coupling constants.

When Φ and $\hat{\Phi}$ acquire VEV's we get the Dirac type neutrino mass terms

$$h_{ij}\overline{\hat{\nu}_{iL}}\nu_{jR} + \frac{v}{\sqrt{2}}\lambda_{ij}\overline{\nu_{iL}}\nu_{jR} + \frac{\hat{v}}{\sqrt{2}}\eta_{ij}\overline{\hat{\nu}_{iL}}\hat{\nu}_{jR} + h.c. \tag{7}$$

which may be written in the form:

$$\overline{\Psi^0_{\nu L}}M^0_\nu\Psi^0_{\nu R} + h.c. \tag{8}$$

with

$$M^0_\nu = \begin{pmatrix} \frac{\mathbf{v}}{\sqrt{\mathbf{2}}}\lambda & \mathbf{0} \\ \mathbf{h} & \frac{\hat{\mathbf{v}}}{\sqrt{\mathbf{2}}}\eta \end{pmatrix} \tag{9}$$

$$\Psi^{0T}_{\nu L} = (\nu_{1L}, \nu_{2L}, \nu_{3L}, \hat{\nu}_{1L}, \hat{\nu}_{2L}, \hat{\nu}_{3L}) \tag{10}$$

$$\Psi^{0T}_{\nu R} = (\nu_{1R}, \nu_{2R}, \nu_{3R}, \hat{\nu}_{1R}, \hat{\nu}_{2R}, \hat{\nu}_{3R}) \tag{11}$$

and $\mathbf{h}$, λ and η are matrices of dimension 3×3.

The standard and mirror neutrino mass matrix M_D is related to the Dirac mass matrix M_ν^0 by a bi-unitary transformation through the equation:

$$U_L^+ M_\nu^0 U_R = M_D \tag{12}$$

where

$$M_D = \begin{pmatrix} m & 0 \\ 0 & \hat{m} \end{pmatrix}, \tag{13}$$

and

$$U_{L,R} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}_{L,R} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & b_{11} & b_{12} & b_{13} \\ a_{21} & a_{22} & a_{23} & b_{21} & b_{22} & b_{23} \\ a_{31} & a_{32} & a_{33} & b_{31} & b_{32} & b_{33} \\ c_{11} & c_{12} & c_{13} & d_{11} & d_{12} & d_{13} \\ c_{21} & c_{22} & c_{23} & d_{21} & d_{22} & d_{23} \\ c_{31} & c_{32} & c_{33} & d_{31} & d_{32} & d_{33} \end{pmatrix}_{L,R} \tag{14}$$

In matrix form we write :

$$\left(\bar{\nu}_L, \bar{\hat{\nu}}_L\right) \begin{pmatrix} \frac{v}{\sqrt{2}}\lambda & 0 \\ h & \frac{\hat{v}}{\sqrt{2}}\eta \end{pmatrix} \begin{pmatrix} \nu_R \\ \hat{\nu}_R \end{pmatrix} \tag{15}$$

Also, in eq.(14), a_{12} is the mixing angle between ν_e and ν_μ, that is, $a_{12} = \theta_{12}$, etc.

As a first approximation we assume the mass matrix in the form

$$M_\nu^0 = \begin{pmatrix} 0 & \frac{\lambda v}{\sqrt{2}} & \frac{\lambda v}{\sqrt{2}} & 0 & 0 & 0 \\ \frac{\lambda v}{\sqrt{2}} & \frac{\lambda v}{\sqrt{2}} & -\frac{\lambda v}{\sqrt{2}} & 0 & 0 & 0 \\ \frac{\lambda v}{\sqrt{2}} & -\frac{\lambda v}{\sqrt{2}} & \frac{\lambda v}{\sqrt{2}} & 0 & 0 & 0 \\ h & h & h & 0 & \frac{\eta \hat{v}}{\sqrt{2}} & \frac{\eta \hat{v}}{\sqrt{2}} \\ h & h & h & \frac{\eta \hat{v}}{\sqrt{2}} & 0 & \frac{\eta \hat{v}}{\sqrt{2}} \\ h & h & h & \frac{\eta \hat{v}}{\sqrt{2}} & \frac{\eta \hat{v}}{\sqrt{2}} & 0 \end{pmatrix} \tag{16}$$

The simple way to find the physical squared neutrino masses is by diagonalization of the mass matrix $M_\nu^0 M_\nu^{0+}$ (or $M_\nu^{0+} M_\nu^0$) using eq.(12). This gives the following eigenvalues:

$$m_1^2 = \lambda^2 v^2, \tag{17}$$

$$m_2^2 = 2\lambda^2 v^2, \tag{18}$$

$$m_3^2 = \frac{1}{2}\left(9h^2 + \lambda^2 v^2 + 2\eta^2 \hat{v}^2 - \sqrt{-16\lambda^2 \eta^2 v^2 \hat{v}^2 + \left(9h^2 + \lambda^2 v^2 + 2\eta^2 \hat{v}^2\right)^2}\right), \tag{19}$$

$$m_4^2 = m_5^2 = \frac{\eta^2 \hat{v}^2}{2}, \tag{20}$$

$$m_6^2 = \frac{1}{2}\left(9h^2 + \lambda^2 v^2 + 2\eta^2 \hat{v}^2 + \sqrt{-16\lambda^2 \eta^2 v^2 \hat{v}^2 + \left(9h^2 + \lambda^2 v^2 + 2\eta^2 \hat{v}^2\right)^2}\right). \tag{21}$$

From here one gets,

$$m_2^2 - m_1^2 \equiv \Delta m_{21}^2 \simeq \lambda^2 v^2 \tag{22}$$

with [8]

$$\Delta m_{21}^2 \simeq 7 \times 10^{-5} eV^2 \tag{23}$$

A more complicated expression can be found for $1.3 \times 10^{-3} eV^2 \leq \Delta_{32}^2 (= m_3^2 - m_2^2) \leq 3.0 \times 10^{-3} eV^2$ [8]. Eq.(22) serves to predict in some sense the $\lambda' s$, $h' s$ and $\eta' s$, so we have:

$$\lambda \simeq 3.4 \times 10^{-14} \tag{24}$$

$$1.8 \times 10^{-2} eV \leq h \leq 2.6 \times 10^{-2} eV \tag{25}$$

Now in the mirror sector it is known [9] that the hadronic decays of right-handed neutrinos, that is $N \rightarrow e^{\pm} + hadrons$, are relevant above the pion threshold, which means $m_N \geq 150 MeV$. On the other hand, a typical mirror scale is $\hat{v} \simeq 10^3 - 10^4 GeV$, which gives:

$$\eta \geq 10^{-4} - 10^{-5} \tag{26}$$

if m_N is generation independent.

For light neutrinos we have the following masses:

$$m_{\nu_1} \simeq 8.4 \times 10^{-3} eV \tag{27}$$

$$m_{\nu_2} \simeq 1.2 \times 10^{-2} eV \tag{28}$$

$$3.8 \times 10^{-2} eV \leq m_{\nu_3} \leq 5.6 \times 10^{-2} eV \tag{29}$$

and for the mirror neutrinos

$$m_{N_1} = m_{N_2} \simeq \frac{\eta \hat{v}}{\sqrt{2}} \tag{30}$$

$$m_{N_3} \simeq \sqrt{2} \eta \hat{v} \tag{31}$$

Taking η of order one we obtain m_N of order $10^3 - 10^4$ GeV.

4 Conclusions

In this paper we have used a left-right symmetric model with two scalar doublets in order to generate neutrino masses. We have written formulae for these masses (standard and mirror) in terms of free and general parameters of the model λ, h and η, as well of the corresponding vacuum expectation values. In first instance one could take the matrix D as zero (meaning absence of mixing among mirror-mirror neutrinos), but due to the full unitarity of the mixing matrix U, this leads us to an also zero matrix A, which is inconsistent since A mixes the standard neutrinos among themselves. Another possibility is to try with a realistic matrix A as dictated by experiment, as worked for instance by King and Mohapatra [10]; however, such choice is not appropriate because although A itself is unitary, it breaks as a consequence the complete unitarity of U.

Finally, we may diagonalise directly the matrix $M^{+}M$ with the help of the mirror scale $\hat{v} \simeq 10^{3} GeV$ (other scales [11] raises $\hat{v}$ up to $10^{10} GeV$) and of the experimental predictions for the squared-neutrino mass differences. In this way we found $\lambda \simeq 10^{-14}$, $\eta \geq 10^{-4} - 10^{-5}$. Needles to say, although we have succeded on predicting these bounds, more effort and data are required to get better information and deep understanding on the content of such parameters, which we hope can help to clarify new physics beyond the Standard Model.

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